Exercise 13

Based on the conditions used to carry out an exothermic reaction in a pilot single-orifice baffle COBR, propose a design for an industrial reactor capable of processing 25 times the pilot feed rate.

Neglect the baffle volume (low baffles thickness).

Pilot COBR data:

Reactor length	L_R	$3\ m$
Baffle spacing	L	$0.045\ m$
Reactor inner diameter	d	$0.025\ m$
Baffle orifice diameter	d_o	$0.01\ m$
Feed flowrate	F	$1.2 \cdot 10^{-5}\ m^3 \cdot s^{-1}$
Frequency	f	$1.6\ Hz$
Center-to-peak amplitude	x_o	$5 \cdot 10^{-3} m$

Physical properties of reaction mixture

$\rho = 840\ kg\ m^{-3}$

$\mu = 5 \cdot 10^{-3}\ Pa \cdot s$

$\lambda = 0.137\ Wm^{-1}K^{-1}$

$c_p = 2150\ J\ kg^{-1}K^{-1}$

Solution

1. ***Pilot-plant key factors***

$d_e = d$ (Single orifice)

$\boldsymbol{\frac{L}{d} = 1.8}$ **(In the optimum range 1.5-1.8)**

$\boldsymbol{\alpha = \left(\frac{d_o}{d}\right)^2 = 0.16}$

$\boldsymbol{Str = \frac{d}{4\,\pi\,x_o} = 0.40}$ **(Slightly below optimal range 0.6-1.7)**

$Re_n = \frac{\rho\,u\,d}{\mu} = 103$

$Re_o = \frac{2\,\pi\,f\,x_o\,\rho\,d}{\mu} = 211$

$\boldsymbol{\psi = \frac{Re_o}{Re_n} = 2.1}$ **(In the optimal range 2-4)**

$\frac{\boldsymbol{P}}{\boldsymbol{V}} = \frac{2\rho N_b}{3\pi C_D^2}\left(\frac{1-\alpha^2}{\alpha^2}\right) x_o^3 (2\pi f)^3 = \boldsymbol{39\ W/m^3}$ assuming $C_D = 0.7$
(x_o *and* f are in the validity range of the equation $5 < x_o < 30$ mm and $0.5 < f < 2$ Hz)

$\boldsymbol{\tau = \frac{V_{pil}}{F_{pil}} = 123\ s}$

$Nu = \frac{hd}{\lambda} = 0.0035 Re_n^{1.3} Pr^{1/3} + 0.3 \left[\frac{Re_o^{2.2}}{(Re_n+800)^{1.25}}\right]$ (Assume valid for the current COBR geometry)

$Nu_{pil} = 14$ (Within validity range 100 < Re_n < 1200 and 0 < Re_o < 800)

$h_{pil} = 77\ Wm^{-2}K^{-1}$

$\left(\frac{A}{V}\right)_{pil} = 160$

$\boldsymbol{\left(h\frac{A}{V}\right)_{pil} = 1.2 \cdot 10^4\ Wm^{-3}K^{-1}}$

2. *Scaleup*

Factors to be kept constant: $\boldsymbol{\tau, \frac{L}{d}, \alpha, Str, \psi}$

Factors that must be at least as high as in the pilot: $\boldsymbol{\frac{P}{V}, h\frac{A}{V}}$

$F_{prod} = 25\ F_{pil}$

$\tau_{prod} = \tau_{pil}$

$V_{prod} = 25\ V_{pil}$

$\frac{L_{prod}}{d_{prod}} = \frac{L_{pil}}{d_{pil}}$, $L_{R,prod} d_{prod}^2 = 25\ L_{R,pil} d_{pil}^2$

From the two latter equations and assuming geometrical similarity $\frac{L_{R,prod}}{d_{prod}} = \frac{L_{R,pil}}{d_{pil}}$, one obtains:

$\rightarrow \boldsymbol{d_{prod}} = d_{pil}\sqrt[3]{25} = \boldsymbol{0.073\ m}$ and $\boldsymbol{L_{R,prod} = 8.8\ m}$

$\rightarrow \boldsymbol{L_{prod}} = \frac{L_{pil}}{d_{pil}} \boldsymbol{d_{prod} = 0.132\ m}$

Note: other relationships between $\frac{L_{R,prod}}{d_{prod}}$ *and* $\frac{L_{R,pil}}{d_{pil}}$ *can be used. For instance, a reduction of the aspect ratio upon scaleup* $\left(\frac{L_{R,prod}}{d_{prod}} < \frac{L_{R,pil}}{d_{pil}}\right)$ *would yield a smaller reactor length. In any case, the impacts on* $\left(h\frac{A}{V}\right), \left(\frac{P}{V}\right)$ *and possibly other variables* $(k_L a, etc\ depending\ on\ the\ type\ of\ process)$ *will have to be verified for each geometry.*

$\alpha_{prod} = \alpha_{pil} = 0.16 \rightarrow \boldsymbol{d_{o,prod}} = d_{prod}\sqrt{0.16} = \boldsymbol{0.029\ m}$

Using $\boldsymbol{f_{prod} = 1.6\ Hz}$ and $\boldsymbol{x_{o,prod} = 0.014\ m}$:

$Re_{n,prod} = 878$

$Re_{o,prod} = 1728$

$\boldsymbol{\psi_{prod} = 2.0}$ (In the right range 2-4 to minimize axial dispersion)

$\boldsymbol{Str_{prod} = 0.42}$ (Slightly below desired range 0.6-1.7 but close to pilot value)

$\boldsymbol{\left(\frac{P}{V}\right)_{prod} = 293\ W/m^3}$ (Significantly higher than pilot value → process performance expected to be at least as high as in the pilot reactor)

$Nu_{prod} = 471$ (But correlation used outside of its validity range 100 < Re_n < 1200 and 0 < Re_o < 800)

$h_{prod} = 8.8 \cdot 10^2\ Wm^{-2}K^{-1}$

$\left(\frac{A}{V}\right)_{prod} = 55$

$\left(h\frac{A}{V}\right)_{prod} = 4.8 \cdot 10^4\ Wm^{-3}K^{-1}$ (Larger than pilot value but correlation for Nu used outside its validity range!)

Note: higher ΔT can be used if $\left(h\frac{A}{V}\right)_{prod} < \left(h\frac{A}{V}\right)_{pil}$